# Novel approach of wavelet analysis for nonlinear ultrasonic measurements and fatigue assessment of jet engine components

Gheorghe Bunget, Brevin Tilmon, Andrew Yee, Dylan Stewart, James Rogers, Matthew Webster, Kevin Farinholt, Fritz Friedersdorf, Marc Pepi, and Anindya Ghoshal

Citation: AIP Conference Proceedings 1949, 070003 (2018); doi: 10.1063/1.5031555

View online: https://doi.org/10.1063/1.5031555

View Table of Contents: http://aip.scitation.org/toc/apc/1949/1

Published by the American Institute of Physics

### Articles you may be interested in

Flaw characterization through nonlinear ultrasonics and wavelet cross-correlation algorithms

AIP Conference Proceedings 1949, 140004 (2018); 10.1063/1.5031609

Nonlinear ultrasonic measurements based on cross-correlation filtering techniques

AIP Conference Proceedings 1806, 060004 (2017); 10.1063/1.4974613

Defect imaging for plate-like structures using diffuse field

The Journal of the Acoustical Society of America 143, EL260 (2018); 10.1121/1.5030915

Forward ultrasonic model validation using wavefield imaging methods

AIP Conference Proceedings 1949, 020012 (2018); 10.1063/1.5031509

Advanced methods in NDE using machine learning approaches

AIP Conference Proceedings 1949, 020022 (2018); 10.1063/1.5031519

Directional nonlinear guided wave mixing: Case study of counter-propagating shear horizontal waves

AIP Conference Proceedings 1949, 070002 (2018); 10.1063/1.5031554



# Novel Approach of Wavelet Analysis for Nonlinear Ultrasonic Measurements and Fatigue Assessment of Jet Engine Components

Gheorghe Bunget <sup>1, a)</sup>, Brevin Tilmon <sup>1</sup>, Andrew Yee <sup>1</sup>, Dylan Stewart <sup>1</sup>, James Rogers <sup>1</sup>, Matthew Webster <sup>2</sup>, Kevin Farinholt <sup>2</sup>, Fritz Friedersdorf <sup>2</sup>, Marc Pepi <sup>3</sup>, Anindya Ghoshal <sup>3</sup>

<sup>1</sup>Murray State University, Engineering Physics Program, Institute of Engineering, Murray, Kentucky
<sup>2</sup>Luna Innovations, Inc., Intelligent Systems Group, Roanoke, Virginia
<sup>3</sup>US Army Research Laboratory, Aberdeen Proving Ground, Maryland

a)Corresponding author: gbunget@murraystate.edu

Abstract. Widespread damage in aging aircraft is becoming an increasing concern as both civil and military fleet operators are extending the service lifetime of their aircraft. Metallic components undergoing variable cyclic loadings eventually fatigue and form dislocations as precursors to ultimate failure. In order to characterize the progression of fatigue damage precursors (DP), the acoustic nonlinearity parameter is measured as the primary indicator. However, using proven standard ultrasonic technology for nonlinear measurements presents limitations for settings outside of the laboratory environment. This paper presents an approach for ultrasonic inspection through automated immersion scanning of hot section engine components where mature ultrasonic technology is used during periodic inspections. Nonlinear ultrasonic measurements were analyzed using wavelet analysis to extract multiple harmonics from the received signals. Measurements indicated strong correlations of nonlinearity coefficients and levels of fatigue in aluminum and Ni-based superalloys. This novel wavelet cross-correlation (WCC) algorithm is a potential technique to scan for fatigue damage precursors and identify critical locations for remaining life prediction.

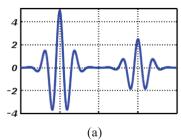
# INTRODUCTION

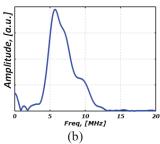
Exposure to significant in-service fluctuations in stress and temperature during repeated ground-air-ground cycles causes turbine engine components to undergo continuous accumulation of DP in the form of crystal dislocations, vacancies, and vacancy cluster that are the major cause of fatigue cracking [1]. It is critical to detect and assess the level of damage precursors before any catastrophic failure occurs. Thus, engine components have to be inspected periodically. Automated nondestructive scanning methods are preferred for inspection due to the complexity and geometric intricacy of the engine components. Amongst various types of nondestructive evaluations (NDE), ultrasonic measurements have proven their effectiveness in achieving a reasonable compromise among resolution, practicality and damage detection. Conventional linear ultrasonic methods based on velocity or attenuation measurements are quite sensitive to macro-defects such as cracks but less sensitive to microstructural changes defined as damage precursors. However, recent decades have seen the emergence of nonlinear ultrasonics (NLU) measurements as effective techniques for tracking the evolution of microstructural changes of fatigued materials. While linear techniques rely on the propagation of ultrasonic vibrations through the medium, the fatigue detection mechanism of nonlinear techniques is based on the interaction between ultrasonic waves and the accumulated damage precursors that distort the waveforms, creating higher order harmonics.

Among the ultrasonic techniques, the pulse-echo mode is preferred where possible to the pulse-through modes because of greater amount of received information, such as the depth of defect location, and system simplicity, since it requires only one probe. However, nonlinear pulse-echo measurements are seldom used to assess fatigue damage. The main reason is that the reflection at stress-free boundary, i.e., backwall interface, causes an unfavorable phase inversion in the second harmonic [2, 3, 4]. However, in practice a number of acoustic phenomena such as beam

diffraction and absorption can contribute to a less perfect cancellation of the second harmonic reflected to the source transducer. These factors were recently proven in the medical field [5, 6, 7, 8, 9], and in the nondestructive field by *Best et al* [10]. It should be mentioned here that the nonlinearity measurement might be inherently less reliable in the pulse-echo mode than those conducted in through-thickness mode. However, by incorporating factors such as absorption and diffraction—Khokhlov-Zabolotskaya-Kuznetsov like model of the sound propagation in solids—the amplitude of the second harmonic can be measured for up to four round trips as was demonstrated by *Best et al* [10].

Another challenge of using NLU measurements on engine components is that their small thickness (turbine blades) requires relatively short-duration sound bursts, making it difficult to extract harmonic amplitudes from the frequency domain. Generally, in the NDE field, the nonlinear generation of second harmonic is limited to relatively long propagation (approx. 10 mm or more) to separate the harmonics and measure their amplitudes [11, 12, 13, 14]. The pulse-inversion technique has been used to cancel the linear fundamental harmonic of the signal and enhance the second harmonic by adding two 180° out-of-phase signals [11, 15]. However, this technique requires sending two successive pulses for each location of the C-scan. This technique is not practical during the continuous motion specific to scanning systems since it requires holding the transducer motionless while transmitting and receiving the two successive tone bursts. Moreover, most ultrasonic scanning systems make use of impulse (usually a negative spike) excitation to generate wideband ultrasonic pulses inside the scanned part since narrow band pulses specific to monochromatic waves are susceptible to noise. The impulse waves make even more difficult the extraction of harmonics since there is no evident separation between them, and the fundamental contribution to the second harmonic may be dominant (Fig. 1).





**FIGURE 1.** Short-duration pulses (a) do not lead to harmonic spacing in frequency domain (b). The second harmonic signal is masked by the fundamental component.

Yet, the feasibility of using nonlinear pulse-echo technique in immersion to inspect thin components was demonstrated by *Kawashima et al* [16, 17] by using a focused transducer for higher harmonics generation. To overcome the problem of scanning thin components, the authors have developed an algorithm based on cross-correlation and wavelet analysis theories to separate the higher order harmonics from a wideband ultrasonic pulse [18]. The objective of current investigation is to develop a nonlinear ultrasonic technique that will be feasible for pulse-echo immersion scanning of hot section aircraft components. Nonlinear C-scan imaging is desirable since it might facilitate the identification fatigue DP build-up. Another objective is to develop a quantitative assessment of the remaining useful life based on the accumulation of fatigue damage precursors. A literature survey as well as some preliminary experiments reported in this paper indicate the potential of measuring the second harmonic of a signal undergoing multiple round trips within the specimen. These experiments have been performed using acoustic harmonic generation through a pulse-echo immersion method. This technique and analysis is proposed in the present paper as a possible NDE tool to study the microscopic internal stresses due to local plasticity effects.

#### TECHNICAL BACKGROUND

To understand the detection mechanism of NLU, let's assume a finite amplitude plane-wave of frequency f that travels through a metallic material. Experimentally, it has been observed that part of the acoustic energy of frequency f (the fundamental component) transfers to its higher harmonics (2f, 3f...) that are generated during wave propagation. This is particularly useful as it enables researchers to detect the early signs of damage build-up in critical components since the transfer of energy to higher harmonic components occurs proportionally with the amount of damage. The Hooke's stress-strain relation of the oscillating material particles can be written in the nonlinear form as a power series expansion [19]

$$\sigma = A\varepsilon + \frac{1}{2}B\varepsilon^2 + \cdots, \tag{1}$$

where  $\sigma$  is the stress created by the propagating wave when oscillating the metallic particles with a displacement gradient, i.e., strain,  $\varepsilon = \partial u/\partial x$ , and A and B are coefficients of the second and third order terms of the power series. Assuming that the wave propagates in the x-direction, the oscillating motion of the metallic particles is described by the wave equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} \,, \tag{2}$$

where  $\rho$  is the material density and  $u = A_1 \sin \omega t$  is the particle displacement when the wave with amplitude  $A_1$  and angular frequency  $\omega$  propagates through the metallic medium. Plugging stress from (1) into (2) and using the displacement gradient, one gets

$$\rho \frac{\partial^2 u}{\partial x^2} = A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2}, \tag{3}$$

This nonlinear differential equation (3) can be solved approximately by iterations [19]. After two iterations, one gets

$$u(x,t) = A_1 \sin(kx - \omega t) - \frac{1}{8} \frac{B}{A} k^2 A_1^2 x \cos[2(kx - \omega t)] + \cdots,$$
 (4)

where  $k = 2\pi/\lambda$  is the wave number,  $\lambda$  is the wavelength of the fundamental amplitude, and x is the propagation distance of the waveform. The second term has the frequency  $2\omega$  and represents the second harmonic of the waveform that is generated by the nonlinearities (inhomogeneities) present in the metallic medium. The amplitude of this second term can be written as [20]

$$A_2 = \frac{1}{8} \left( \frac{B}{A} \right) A_1^2 k^2 x,\tag{5}$$

The term B/A in relation (5) represents the known nonlinearity parameter,  $\beta$ , and rearranging the terms its known form is

$$\beta = \frac{A_2}{A_1^2} (\frac{8}{k^2 x}),\tag{6}$$

When applied to longitudinal waves in isotropic materials, *Breazeale et al* [21, 22] related the nonlinearity parameter to other second and third order material elastic constants such as, Lamé  $(\lambda, \mu)$  and Murnaghan (l, m) constants

$$\beta = \frac{3}{2} + \frac{l+2m}{\lambda + 2\mu},\tag{7}$$

The nonlinearity parameter can be calculated based on relation (6) from experimental measurements of harmonic amplitudes,  $A_1$  and  $A_2$ . Since the sound velocity of compressional waves is independent of frequency of the wave and, when the propagation distance is constant, one may use only the electrical amplitudes to perform a relative measurement of the nonlinearity parameter

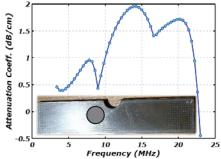
$$\beta \propto \beta' = \frac{A_2'}{\left(A_1'\right)^2},\tag{8}$$

#### MEASUREMENTS AND METHODS

When the frequency of the applied ultrasound is near the resonance frequency of DP, they absorb the ultrasonic energy effectively. Therefore, by determining where the local maxima of the attenuation coefficient of the fundamental amplitude occurs in the frequency domain, optimal detection of DP is obtained even at low power excitation. For example, by using three broadband contact transducers with central frequencies at 5, 10, and 20 MHz (Olympus, models V201, V202, and M2017) and a wideband pulser/receiver (JSR, model DPR300) in pulse echo mode, three maxima of absorption coefficient were obtained (Fig. 2). These measurements were performed on a 95% fatigued specimen of single crystal Ni-base superalloy, at the location where accumulation of DP was expected. Local maxima were obtained for each transducer: 7.7 for 5 MHz, 14 for 10 MHz, and 20 for 20 MHz. However, the attenuation coefficient had a global maxima for this material at approximately 14 MHz, even though one would expect a higher attenuation for higher frequency. This aspect needs to be explored further, but it is not investigated here.

# **Specimens Preparation**

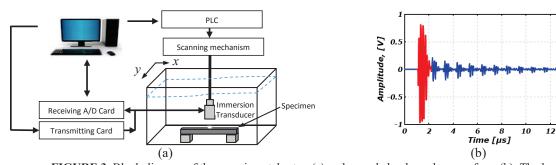
Flat tensile test specimens of a Ni-based single-crystal superalloy were prepared with dimensions  $25 \times 3.18 \times 124$  mm (Fig. 2). The thickness of the specimens (3.18 mm) was selected to be representative of thin turbine blades. A round notch was machined via electro-discharge machining to concentrate plastic zones around it. Each specimen was mirror polished to a roughness Ra  $\approx 0.05~\mu m$  to remove manufacturing defects and ensure consistent surface roughness across the entire library of specimens. Increases in the dislocation density of the single crystal specimens were obtained through cyclical tension-tension stress controlled loadings in an MTS system with a loading rate of 10 cycles per second. Maximum stress was  $\sigma_{max} = 345$  MPa and the stress ratio was R = 0.1. Destructive fatigue tests were performed on three specimens to estimate the average number of cycles to failure for this selected cyclic stress. Next, four increasing levels of fatigue -0, 25, 62.5, and 95% – were achieved.



**FIGURE 2.** Attenuation coefficient vs. frequency for the most fatigued specimen of Ni-base superalloy showing the measurement location.

# Pulse-Echo Nonlinear C-Scan Ultrasonic Measurements

A scanning system (MISTRAS, model UPK36) was used in pulse-echo mode to scan each specimen in the library of Ni-base superalloys (Fig. 3). Its pulser, however, could not perform at the global maximum of 14 MHz identified previously, because its limited bandwidth could not capture the second harmonic of 28 MHz (Fig. 2). Therefore, the first resonant peak of 7 MHz was selected to excite a broadband flat transducer with central frequency of 10 MHz (Olympus, model A312-S). Initially, a set of measurements and C-scans were performed on an existing dogbone specimen of Inconel 718 having dimensions 25 × 6.35 × 215 mm that was failed through low cycle fatigue (Fig. 4a). These preliminary measurements were performed to assess the capability of the measurement method on a specimen having large amounts of DP due to plastic deformation. These scans were performed with a spatial resolution of 0.5 mm and long waveforms, i.e., up to 11 backwall echoes, were recorded. Because the amplitude of the second harmonic is smaller than the fundamental amplitude, its attenuation in water and the studied material could yield an unclear signal. In this case, the transducer was located approximately 20 mm from the actual front surface of the specimen.



**FIGURE 3.** Block diagram of the experimental setup (a) and recorded pulse-echo waveform (b). The large red pulse represents the front surface echo, which could be used for corrections of edge effects.

# RESULTS AND DISCUSSION

The authors have developed a wavelet cross-correlation (WCC) algorithm to separate and extract harmonic amplitudes from the otherwise wideband signal [18]. DP were detected in aluminum and Inconel 718 specimens by using WCC algorithm in pulse-through mode, both in contact and in immersion. Yet, the second harmonic was in the noise for contact pulse-echo mode and it could not be used for excitations with negative pulse conventional pulser/receivers. For this current investigation, two modifications were introduced in the WCC algorithm: (i) the number of cycles of the template signal was increased four times, and (ii) no windowing was applied to the sinusoidal wavelet of the template signal. Moreover, these measurements were performed using a pulser/receiver with the capability of generating tone bursts.

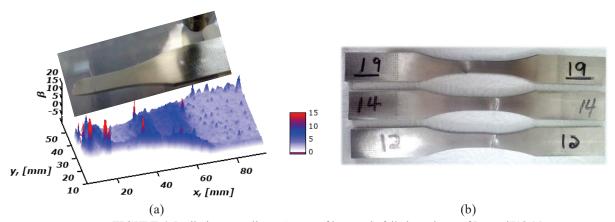
The numerical analysis of each waveform was performed in Octave (open source MATLAB-like software). The amplitudes of the fundamental,  $A'_{l}$ , and second harmonic,  $A'_{2}$ , were extracted from each of the reflected back wall echoes via the improved WCC algorithm. A relative nonlinearity parameter was calculated as a cumulative weighted average of the harmonic amplitudes of each back wall echo using the relation

$$\beta_{rel} = \frac{\sum_{i=1}^{n} \frac{A_2'}{2 \cdot x \cdot i}}{\sum_{i=1}^{n} (A_1')^2},\tag{9}$$

where  $A'_1$ ,  $A'_2$ , are the electric relative amplitudes not representing the absolute oscillations of the metallic particles, and x was the thickness of each specimen calculated based on the sound velocity measured for each waveform in the C-scan.

A nonlinear C-scan image was constructed from all recorded waveforms by using the nonlinearity parameter,  $\beta$ , calculated with relation (9). Large values of  $\beta$  were observed at the fracture zone and at the small crack that was nucleated at one edge of the Inconel specimen (Fig. 4a). Then,  $\beta_{rel}$  decreased significantly to values of approx. 3 – 3.5 in the gauge section between x = 25 mm and x = 40 mm. Starting at x = 40 mm,  $\beta_{rel}$  again exhibited a significant increase to values of 7.5 – 8 in the transition zone and then, decreased to values of 3 – 3.5 in the gripping section. It is important to mention here that all three specimens of Inconel 718 within the sample population that were fatigued to failure, failed at the beginning of the transition zones (Fig. 4b).

It appears that the nonlinear ultrasonic measurements indicate plastic zones of high DP densities at transition sections of the specimens, but any further attempts at analysis would be pure conjecture unless micrographic analysis is carried out on these specimens. Yet, this nonlinear C-scan in pulse echo mode was in agreement with prior nonlinear pulse-through measurements performed on the same Inconel specimen [18]. Therefore, these results deserve further investigation based on metallographic analysis. It was noticeable that the surface roughness did not negatively affect the waveform analysis with the WCC algorithm and nonlinear measurements. The specimen gauge and transition sections were mirror polished, but the gripping sections had visible manufacturing surface texture (Ra  $\approx 1.6 \,\mu m$ ).



**FIGURE 4.** Preliminary nonlinear C-scan of low-cycle failed specimen of Inconel718 (a). Tested specimens of Inconel 718 failed at the transition zone (b).

After seeing that the analysis was suitable for the Inconel 718 specimen, it was carried out on the single crystal specimens. A steady increase in the second harmonic amplitude,  $A'_2$ , was observed with increasing fatigue level even though the first harmonic,  $A'_1$ , exhibited only a slight increase between the pristine and 25% fatigued specimen (Fig. 5). The second harmonic showed a significant rounded maximum for the 95% fatigued specimen. This peak was asymmetrically located with respect to the notch. A fine crack, approximately 4 mm long, was observed in a preliminary micrographic analysis (Fig. 6a). This appreciable increase of the second harmonic can be explained in terms of dislocation density occurring in the plastic zone formed around the tip of the crack. On the other hand, linear conventional ultrasonic measurements—the amplitude of the first backwall echo—was not sensitive to DP (Fig. 7). Only the crack at the rounded notch could be visible in the conventional C-scan.

Yet, starting with the fourth backwall echo, the second harmonic exhibits a noisy behavior indicating that its values may be less reliable. This is in agreement with the results obtained by *Best et al* [10] with the conclusion that detectable nonlinearity was possible at least up until the third and fourth reflection. However, by correcting the nonlinearity parameter calculated with relation (9) for the attenuation effects, stable nonlinearity parameters could be used up to the sixth reflection. This correction was made by using the relation [23]

$$\beta = \beta_{meas.} \frac{\alpha_2 - 2\alpha_1}{1 - \exp[(2\alpha_1 - \alpha_2) \cdot x]},\tag{10}$$

where  $\alpha_1$ ,  $\alpha_2$  are the attenuation coefficients of the two harmonics,  $A'_1$  and  $A'_2$ . The attenuation of the second harmonic exhibited an increase with the fatigue level indicated by the continuous lines in Fig. 6b.

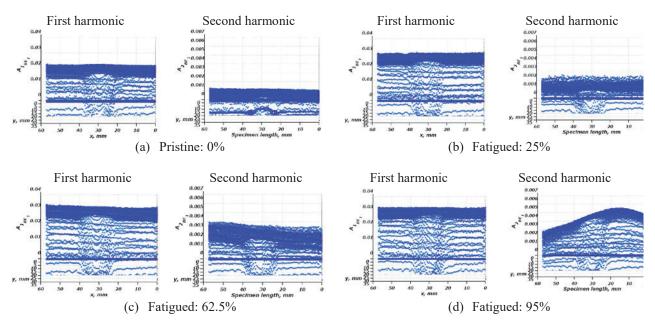
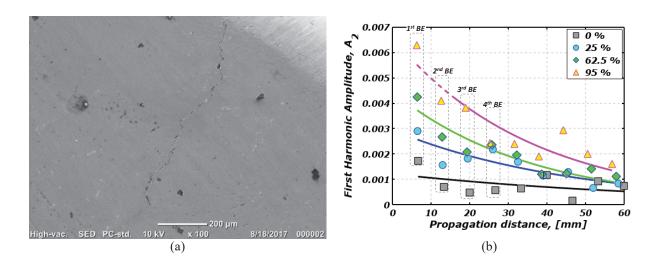
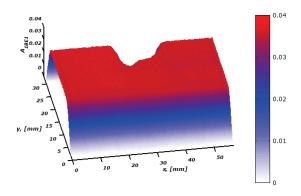


FIGURE 5. The second harmonic of the first backwall increased uniformly with the fatigue level, while the first harmonic remained relatively constant.

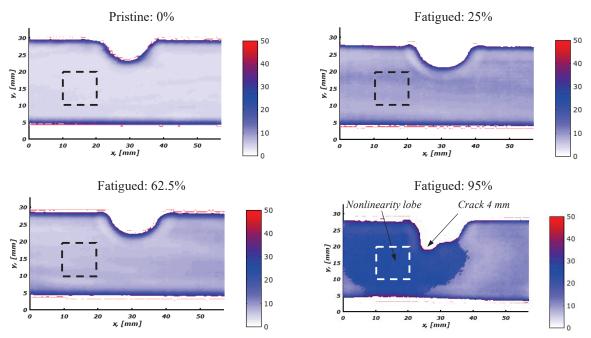


**FIGURE 6.** Metallographic image indicating a fine surface crack at the notch of the 95% fatigued specimen (a). The amplitude of the second harmonic, Az, increased significantly with fatigue level (b).



**FIGURE 7.** The C-scan based on conventional ultrasonic measurements on the 95% fatigued specimen shows only the flaw on the left of the rounded notch. No indication of DP is visible through these linear ultrasonic measurements.

As series of nonlinear C-scan images were produced by using relation (10). Color intensity increases uniformly with fatigue level (Fig. 8). A lobe of intense color surrounds the tip of the crack specimen with 95% fatigue life. The authors believe that this lobe is related to the plastic region known as the crack tip plastic zone. The ductile crack growth is accompanied by the expansion of a plastic zone based on decohesion or fracture of inclusions. It is known that when the crystal is plastically deformed, dislocation density will increase, which will lead to an increase in the nonlinearity parameter,  $\beta$  [24, 25].



**FIGURE 8.** Nonlinear C-scan images of four specimens of single crystal Ni base superalloy indicating a continuous increase of the nonlinearity parameter. A square region from each specimen was selected for quantitative analysis.

A finite element analysis (FEA) was performed to qualitatively assess which locations undergo the largest stresses during tensile testing (Fig. 9). The simulation was conducted using FEMAP software with the geometry and boundary condition representative to a linear quasi-static tensile test. This simulation was run using hexagonal elements with the maximum Von-Mises stress calculated over the test section. Two lobes of high stress radiating from an angle of approximately 45° on either side of the stress concentrator notch are observed. While the stress distribution here does not necessarily correlate directly to plastic deformation zones in test specimens due to the influence of crystal orientation, it is likely that similar patterns will be identified from NLU C-scans of the specimens.

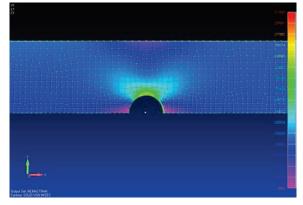


FIGURE 9. FEA modeling indicating stress distribution lobes at the rounded notch.

A set of waveforms that are representative to sensitive regions of high stress around the notch in the nonlinear C-scan were selected (dashed square) for a quantitative comparison of the nonlinearity parameter  $\beta$ . The nonlinearity parameter increases monotonically by hundreds percent over the fatigue life of the material (Fig. 10). The relative values of  $\beta$  were estimated at the level of  $\pm 4\%$  experimental uncertainty. The harmonic amplitudes of the first five reflections were used to calculate the nonlinearity parameters in this present paper.

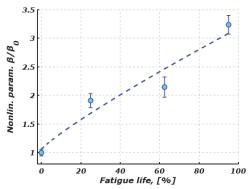


FIGURE 10. Quantitative analysis of nonlinearity parameter in the selected regions at rounded notches.

# **CONCLUSIONS AND FUTURE WORK**

The feasibility of using pulse-echo immersion method was investigated as a practical technique of sensing damage precursors (DP), mapping nonlinear C-scan images, and estimating the remaining useful life of inspected components. Harmonic generation from a failed specimen of Inconel 718 and several specimens of Ni-base superalloy with increasing levels of fatigue damage was measured based on an improved wavelet cross-correlation (WCC) algorithm. The harmonic generation was favored by selecting an appropriate excitation frequency that was near the resonance frequency of DP. The hypothesis made here was that DP would absorb ultrasonic energy effectively at their resonant frequency and generate higher harmonics even though the incident energy was reduced. The immersion nonlinear measurements presented here indicate that less perfect cancellations of the second harmonic reflected to the source transducer are possible up to the fourth and even fifth reflection. These results are in agreement with prior results obtained by other researchers in the medical and NDE fields.

The uniqueness of present investigation lies in the authors' efforts to overcome the aforementioned challenges of pulse-echo nonlinear measurements by developing a harmonics detection algorithm. The WCC algorithm indicates significant potential for extracting the harmonics present in a non-amplified ultrasonic waveform. The authors consider that the WCC algorithm makes the use of standard instrumentation (without power amplification) a viable solution for nonlinear ultrasonics. The immersion measurements and presented data analysis demonstrated a promising technique for mapping damage precursors in nonlinear C-scan imaging. This claim, however, needs to be corroborated by metallographic analysis of damage precursors, and relevant work in this direction is considered a part of the authors' future work.

# **ACKNOWLEDGMENTS**

The authors are thankful to Prof. Peter Nagy and Laurence Jacobs for bringing their concerns and suggestions to the attention of the authors. The research reported in this document/presentation was performed in connection with contract/instrument W911QX-13-C-0162 with the U.S. Army Research Laboratory. The views and conclusions contained in this document/presentation are those of the authors and should not be interpreted as presenting the official policies or position, either expressed or implied, of the U.S. Army Research Laboratory or the U.S. Government unless so designated by other authorized documents. Citation of manufacturer's or trade names does not constitute an official endorsement or approval of the use thereof. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

#### REFERENCES

- [1] T. Grobstein, S. Sivashankaran, G. Welsh, N. Panigrahi, J. McGervey and J. Blue, "Fatigue damage accumulation in nickel prior to crack initiation," *Mat. Sci. Engr.*, **A138**, 191-203, (1991).
- [2] R. Fay, "Oppositely Directed Plane Finite Waves," J. Acoust. Soc. Am., 29:11, 1200 1203, (1957).
- [3] M. Breazeale and W. Lester, "Demonstration of the "Least Stable Waveform" of Finite Amplitude Waves," *J. Acoust. Soc. of Am.*, **33**:12, 1803, (1961).
- [4] D. Thompson, M. Tennison and O. Buck, "Reflections of Harmnonic Generation by Finite-Amplitude Waves," *J Acoust Soc Am*, **44**:10, 435 6, (1968).

- [5] M. Nikoonahad and D. Liu, "Pulse-echo Single Frequency Acoustic Nonlinearity Parameter (B/A) Measurement," *IEEE Trans. Ultras. Ferroelec. & Freq. Control*, **37**:2, 127, (1990).
- [6] F. Meulen and L. Haumesser, "Evaluation of B/A nonlinear parameter using an acoustic self-calibrated pulseecho method," *Appl. Phys. Lett.*, 92, (2008).
- [7] S. Saito, J. Kim and K. Nakamura, "Automatic measurement of the nonlinearity parameter B/A in liquid media," *Ultrasonics*, 44, 1429 1433, (2006).
- [8] A. Voronovich, "On the conservation of momentum for a sound pulse reflecting from a pressure-release boundary," *J Acoust Soc Am*, **123**:5, 2480-3, (2008).
- [9] D. Kourtiche, L. Allies, A. Chitnalah and M. Nadi, "Harmonic propagation of finite amplitude sound beams: comparative method in pulse echo measurement of nonlinear B/A parameter," *Meas Sci Technol*, 12,1990-5, (2001).
- [10] S. Best, A. Croxford and S. Neild, "Pulse-Echo Harmonic Generation Measurements for Non-destructive Evaluation," *J. Nondestructive Eval*, **33**, pp. 205-215, (2014).
- [11] Y. Ohara, K. Kawashima, R. Yamada and H. Horio, "Evaluation of amorphous diffusion bonding by nonlinear ultrasonic method," Review of Progress QNDE (Vol. 23), *AIP Conf. Proc.*, 944-951, (2004).
- [12] J. Frouin, T. Matikas, J. Na and S. Sathish, "In-Situ Monitoring of Acoustic Linear and Nonlinear Behavior of Titanium Alloys during Cycling Loading," *Proc. of SPIE Conf. on NDE of Aging Materials*, Newport Beach, California, 3585 (1999).
- [13] C. Pruell, J. Kim, J. Qu and L. Jacobs, "Evaluation of fatigue damage using nonlinear guided waves," *Smart Mater Struct*, **18**(3) 035003 (2009).
- [14] A. Viswanath, B. Rao, S. Mahadevan, P. Parameswaran, T. Jayajumar and B. Raj, "Nondestructive assessment of tensile properties of cold worked AISI type 304 stainless steel using nonlinear ultrasonic technique," *J Mat Proc Tech*, 211, 538-544, (2011).
- [15] J. Kim, L. Jacobs, J. Qu and J. Littles, "Experimental characterization of fatigue damage in nickel-base superalloy using nonlinear ultrasonic waves," *J. Acoust. Soc. Am.*, **12**0, 1266, (2006).
- [16] K. Kawashima, R. Imanishi, F. Fujita and T. Aida, "Imaging of Partial Plastic Deformation in Thin Metal Plates by Immersion Nonlinear Ulrasonic Local resonance," *AIP Conf. Proc. Nonlinear Acoustics*, Tokio, Japan, pp. 207-210 (2012).
- [17] K. Kawashima, T. Aida and H. Yasui, "Higher-harmonic imaging of the plastic zone in front of a fatigue crack tip," *Jap J Appl Phys*, **53**, (2014).
- [18] A. Yee, D. Stewart, G. Bunget, P. Kramer, K. Farinholt, F. Friedersdorf, M. Pepi and A. Ghoshal, "Nonlinear ultrasonics measurements based on cross-correlation filtering techniques," Review of Progress in QNDE (Vol. 36), *AIP Conference Proceedings* **1806**, 060004, Atlanta, Georgia, (2017).
- [19] A. Hikata, B. Chick and C. Elbaum, "Dislocation Contribution to the Second Harmonic Generation of Ultrasonic Waves," *J. Appl. Phys.*, **36**, 229, (1965).
- [20] J. Cantrell, "Generalized Gruneisen tensor from solid nonlinearity parameters," *Phys Rev B*, **21**:10, 4191-5, (1980).
- [21] D. Joharapurkar and M. Breazeale, "Nonlinear parameter, nonlinearity constant, and frequency dependence of ultrasonic attenuation in GaAs," *J Appl Phys*, **67**:1, 76-80, (1990).
- [22] D. Joharapurkar, D. Gerlich and M. Breazeale, "Temperature dependence of elastic nonlinearities in single-crystal gallium arsenide," *J Appl Phys*, **72**:6, 2002-8, (1992).
- [23] H. Mohrbacher, D. Lee, E. Schneider and K. Salama, "Acoustic nonlinearity in metal-matrix composites," *Review of Prog. in QNDE*, **10B**, pp. 1821-1828, (eds) D.O. Thompson and D.E. Chimenti, Plenum Press, New York (1991).
- [24] A. Hikata and C. Elbaum, "Generation of Ultrasonic Second and Third Harmonics Due to Dislocations. I," *Phys Rev*, **144**:2, 469-77, (1966).
- [25] A. Hikata, F. Sewell and C. Elbaum, "Generation of Ultrasonic Second and Third Harmonics due to Dislocations. II," *Physical Rev*, **151**:2, 442-9, (1966).